Causality Checking for Complex System Models

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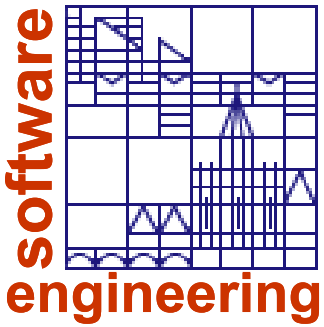

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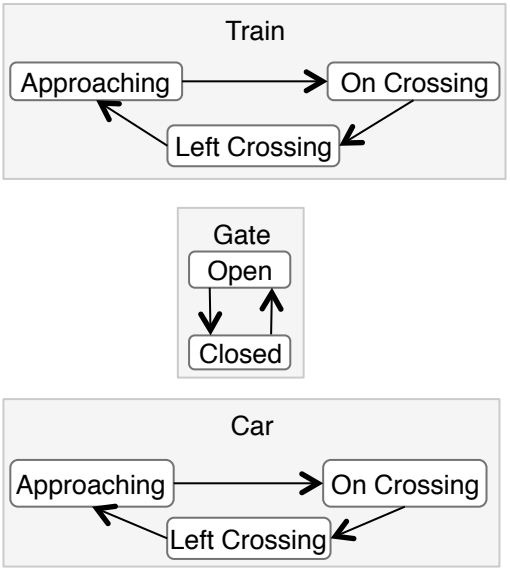
Analysis of Complex Systems

- **A Railroad Crossing**

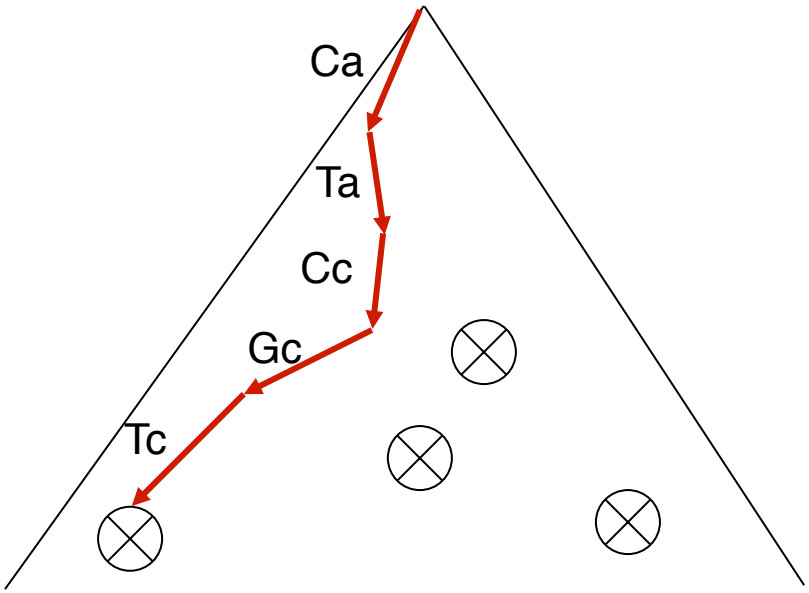
Model Checking

$M \models S$

model of the software (transition system, Kripke structure)	model checking algorithm	requirement specification (assertions, temporal logic, automata)
	state space search (depth-first or breadth-first search)	*there is never a train in the crossing at the same time when there is a car in the crossing* $\varphi = \Box\neg(Tc \wedge Cc)$

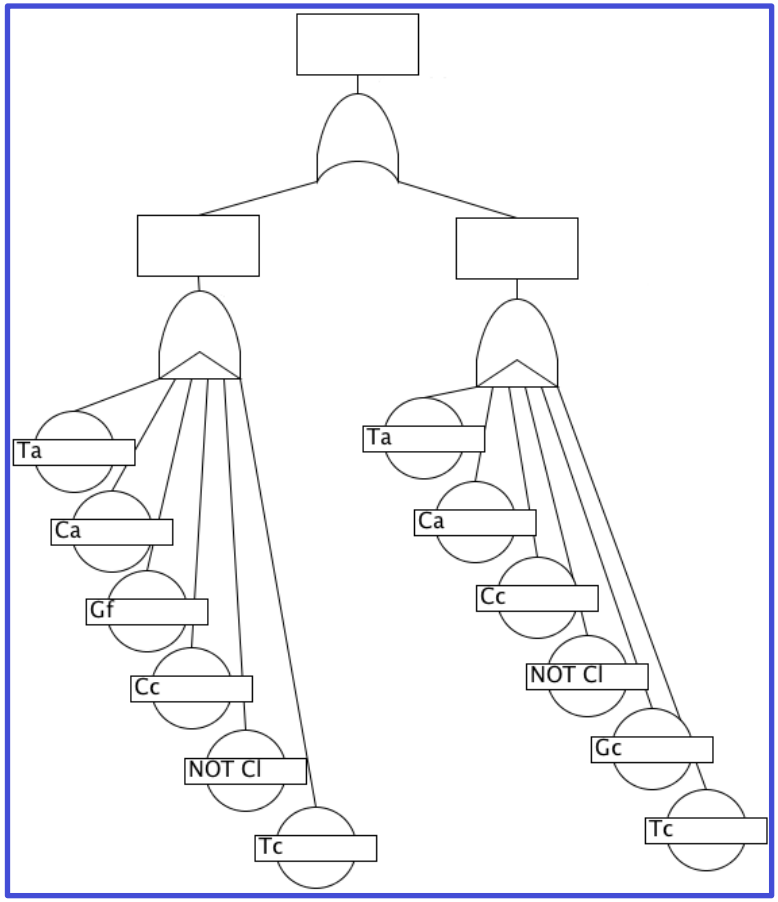
Model Checking

- **Explicit State Model Checking**
  - most common: automatic search of all reachable system states to find property violations using depth-first search (DFS) or breadth-first search (BFS)
  - the path into a property violating state is called an error path or counterexample



Interpreting Counterexamples

- **Railroad Crossing Example:**
  - 11 error-paths (only considering shortest paths)



- all lead into a property violating state (accident)
- for debugging
  - **what is the cause?**
- manual analysis
  - tedious
  - error prone
  - essentially impossible
- our goal:
  - **algorithmic causality computation**

Outline

- Models of Causation
- An Adopted Structural Equation Model
- Causality Checking
- Experimental Evaluation
- Conclusion

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Causality

- **(Naive) Lewis Counterfactual Reasoning**

  c is causal for e (effect / hazard) if, had c not happened, then e would not have happened either

  - logical foundation of some software debugging techniques, e.g.,
    - delta debugging
    - nearest neighbor techniques
  - best suited for single cause failures

Consequences

- **Need for Alternate Worlds**
  - what-if analysis
    - had there been another course of action (= "world") in which the gate had been closed before the car entered the crossing, there would not have been an accident (= a "good" world)
  - "good" world: the effect does not occur
  - "bad" world: the effect occurs

- **Limitations**
  - not suited for effects that have logically complex causal structure
  - We use an adaption of the **Structural Equation Model** by Halpern and Pearl
    - SEM is based on Lewis counterfactional reasoning

Halpern / Pearl Structural Equation Model (SEM)

- **Key Ideas**
  - events are represented by boolean variables
    - specified using structural equations
  - computes minimal boolean disjunction and conjunction of causal events
  - causal dependency of events represented by causal networks
  - reference

J. Halpern and J. Pearl, "Causes and explanations: A structural-model approach. Part I: Causes," *The British Journal for the Philosophy of Science*, 2005.

Halpern / Pearl Structural Equation Model (SEM)

- **Actual Causality Conditions**
  - AC1: ensures that there exists a world where the boolean combination of causal events **c** and the effect **e** occur
  - AC2:
    1. if at least one of the causal events does not happen, the effect **e** does not happen
    2. if the causal events occur, the occurrence of other events can not prevent the effect
  - AC3: no subset of the causal events satisfies AC1 and AC2 (minimality)

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An Adopted Structural Equation Model

- **Main Goals**
  - Consider event order as causal factor
  - Make Structural Equation Model applicable to transition systems

Event Order Logic

- **Boolean Event Occurrence Conditions**
  - $a \wedge b$, $a \vee b$, $\neg a$
- **Event Ordering Conditions**
  - $a \wedge\!\!\cdot\, b$
    - a and b occur, and a occurs before b
- **Interval Operators**
  - $a \wedge\!\!\cdot\, [\, b$
    - a occurs until eventually b will hold in every state
  - $a \wedge\!\!\cdot\,]\, b$
    - a always holds until eventually b occurs
  - $a \wedge\!\!\cdot_{<} b \wedge\!\!\cdot_{>} c$
    - in the interval delimited by a and c, b always holds
- **Model-theoretic Semantics**
  - Event Order Logic is an LTL

Event Order Logic

- **Representation of Traces**

$$\sigma = \text{“Ta, Ca, Gf, Cc, Tc”}$$

$$\psi = \mathrm{Ta} \mathbin{\dot{\wedge}} \mathrm{Ca} \mathbin{\dot{\wedge}} \mathrm{Gf} \mathbin{\dot{\wedge}} \mathrm{Cc} \mathbin{\dot{\wedge}} \mathrm{Tc}$$

- **Representation of Ordering Constraints**

$$(\mathrm{Ta} \wedge \mathrm{Ca}) \mathbin{\dot{\wedge}} \mathrm{Gf}$$

$$\mathrm{Cc} \mathbin{\dot{\wedge}}_{<} \neg \mathrm{Cl} \mathbin{\dot{\wedge}}_{>} \mathrm{Tc}$$

Adopted SEM

- Is $\psi = \mathrm{Ta} \wedge \mathrm{Ca} \wedge \mathrm{Gf} \wedge \mathrm{Cc} \wedge \mathrm{Tc}$
  causal for the violation of $\varphi = \Box\neg(\mathrm{Tc} \wedge \mathrm{Cc})$?

- **AC1**
  - there exists σ so that both $\sigma \models \psi$ and $\sigma \not\models \neg\varphi$

- **Remarks**
  - the "positive" side of counterfactual test
  - True if there exists a error-path σ = Ta, Ca, Gf, Cc, Tc

Adopted SEM

- Is $\psi = \mathrm{Ta} \wedge \mathrm{Ca} \wedge \mathrm{Gf} \wedge \mathrm{Cc} \wedge \mathrm{Tc}$ causal for the violation of $\varphi = \Box\neg(\mathrm{Tc} \wedge \mathrm{Cc})$?

- **AC2 (1)**
  - $\exists\ \sigma'$ where the order and occurrence of events is different from σ and φ is not violated on σ'

- **Remarks**
  - this is the counterfactual test
  - AC2 (1) fulfilled by $\psi = \mathrm{Ta} \wedge \mathrm{Ca} \wedge \mathrm{Gf} \wedge \mathrm{Cc} \wedge \mathrm{Tc}$
    - since there exists σ' = Ta, Ca, Gc, Tc so that
      - σ' is different from σ = Ta, Ca, Gf, Cc, Tc
      - $\sigma' \models \varphi$ (σ' is a "good" path)

Adopted SEM

- Is $\psi = \mathrm{Ta} \wedge \mathrm{Ca} \wedge \mathrm{Gf} \wedge \mathrm{Cc} \wedge \mathrm{Tc}$ causal for the violation of $\varphi = \Box\neg(\mathrm{Tc} \wedge \mathrm{Cc})$?

- **AC2 (2)**
  - for a sequence of events to be causal it cannot be possible to add an event so that causality is voided

- **Remarks (1)**
  - Motivation
    - serves to reveal that non-occurrence is causal
  - consider σ" = "Ta, Ca, Gf, Cc, Cl, Tc"
    - for σ" the property φ is not violated since Cl occurs before Tc
  - consequence:
    - ψ is not causal (AC2 (2) fails)
    - the non-occurrence of an event (Cl) is causal

Adopted SEM

- Is $\psi = \mathrm{Ta} \barwedge \mathrm{Ca} \barwedge \mathrm{Gf} \barwedge \mathrm{Cc} \barwedge \mathrm{Tc}$
  causal for the violation of φ = □¬(Tc ∧Cc) ?

- **Causality of Non-Occurrence (what if AC2(2) fails?)**
  - steps
    - find minimal set of causal non-occurrence events
    - add, depending on the position of the event in a_α this set
      - $\neg a_\alpha \barwedge]$ at the beginning of ψ
      - $\barwedge_[\neg a_\alpha$ at the end of ψ
      - $\barwedge_< \neg a_\alpha \barwedge_>$ in the middle of ψ
      - perform test AC2 (2) again
  - example

$$\psi = \mathrm{Ta} \barwedge \mathrm{Ca} \barwedge \mathrm{Gf} \barwedge \mathrm{Cc} \barwedge_< \neg \mathrm{Cl} \barwedge_> \mathrm{Tc}$$

Adopted SEM

- Is $\psi = \mathrm{Ta} \wedge \mathrm{Ca} \wedge \mathrm{Gf} \wedge \mathrm{Cc} \wedge \mathrm{Tc}$
  causal for the violation of $\varphi = \Box\neg(\mathrm{Tc} \wedge \mathrm{Cc})$?

- **AC3**
  - ψ is minimal: no subset of ψ satisfies conditions AC1 and AC2

Adopted SEM

- Is $\psi = \mathrm{Ta} \mathbin{\dot{\wedge}} \mathrm{Ca} \mathbin{\dot{\wedge}} \mathrm{Gf} \mathbin{\dot{\wedge}} \mathrm{Cc} \mathbin{\dot{\wedge}} \mathrm{Tc}$ causal for the violation of $\varphi = \Box\neg(\mathrm{Tc} \wedge \mathrm{Cc})$?

- **OC1 Causality of Event Order**
  - let Ψ an eol formula over some events in ψ
  - for some eol formula Ψ, replace the ordered operator $\dot{\wedge}$ by the unordered $\wedge$ yielding $\Psi_{\wedge}$
  - the order expressed by Ψ is not causal if
    - $\sigma \vDash \Psi \wedge \exists \sigma' \in \Sigma_B : \sigma' \nvDash \Psi \wedge \sigma' \vDash \Psi_{\wedge}$
  - example
    - order of events Gf, Cc, ¬Cl, Tc is important for causing property violation
    - relative order of Ta and Ca is not important, but they need to precede the above events
    - resulting formula $\psi = (\mathrm{Ta} \wedge \mathrm{Ca}) \mathbin{\dot{\wedge}} \mathrm{Gf} \mathbin{\dot{\wedge}} \mathrm{Cc} \mathbin{\dot{\wedge}}_{<} \neg\mathrm{Cl} \mathbin{\dot{\wedge}}_{>} \mathrm{Tc}$

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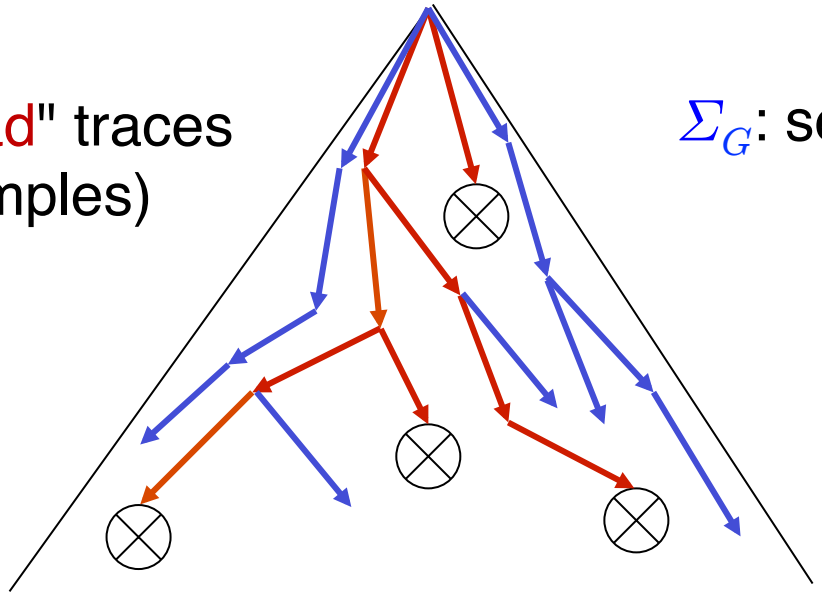
Execution Traces and Counterfactuals

- **Traces Define (Alternate) Worlds**

- **Computed by State Space Search**
  - model checking
    - traverse state space using BFS or DFS
  - applicable to reachability properties
    - no meaningful behavior after property violation is observed

Σ_B: set of "bad" traces (counterexamples)



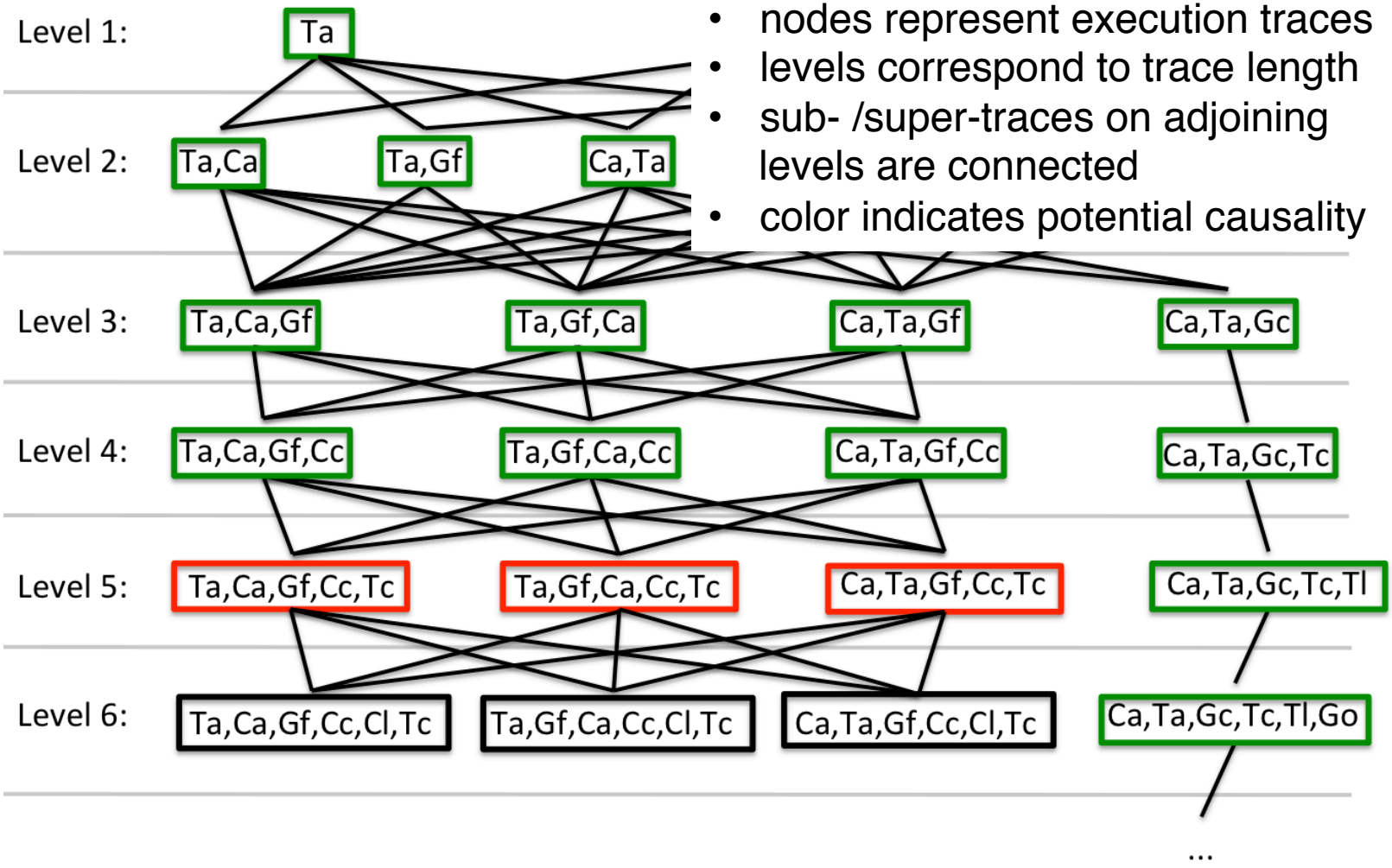
Σ_G: set of "good" traces

Algorithmics

- **Sub-Executions**
  - reduce checks for AC1-AC3 and OC1 to sub-execution tests
    - ordered and unordered sub-execution operators
  - proofs in the paper

- **Implementation Variants**
  - Off-line Enumeration
    - enumerate traces
    - store Σ_B and Σ_G
    - perform sub-trace computations
  - On-the-fly
    - use DFS / BFS on the state space
      - store paths in an adequate data structure as you obtain them
        - subset graph

Subset Graph



- nodes represent execution traces
- levels correspond to trace length
- sub- /super-traces on adjoining levels are connected
- color indicates potential causality

Inference From Subset Graph

- **Theorems for Adopted SEM Conditions**
  - eol formula ψ_σ derived from a red node σ fulfills AC1 and AC2(1) and AC3
    - for BFS: AC3 fulfilled immediately
    - for DFS: when search terminates

- **Construction of Subset Graph**
  - on the fly during state space search
  - once state space search complete, perform tests for AC2(2) and OC1

Complexity (Preliminary)

- **Caveat**
  - *even for an SEM with only binary variables, computing causal relationships between variables is NP-complete*

    Eiter and Lukasziewicz, 2002

- **However**
  - Eiter and Lukasziewicz, 2006: for cycle-free causal dependencies computing causal relationships can be done polynomially
  - naturally, this is only part of the complexity analysis…

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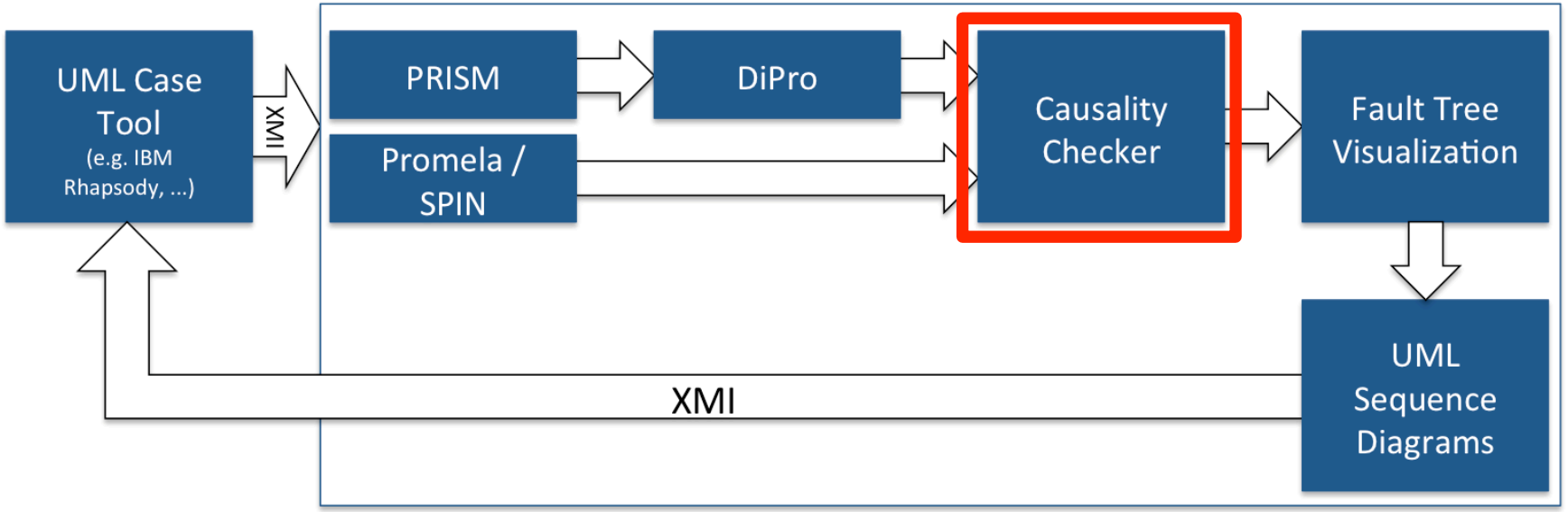
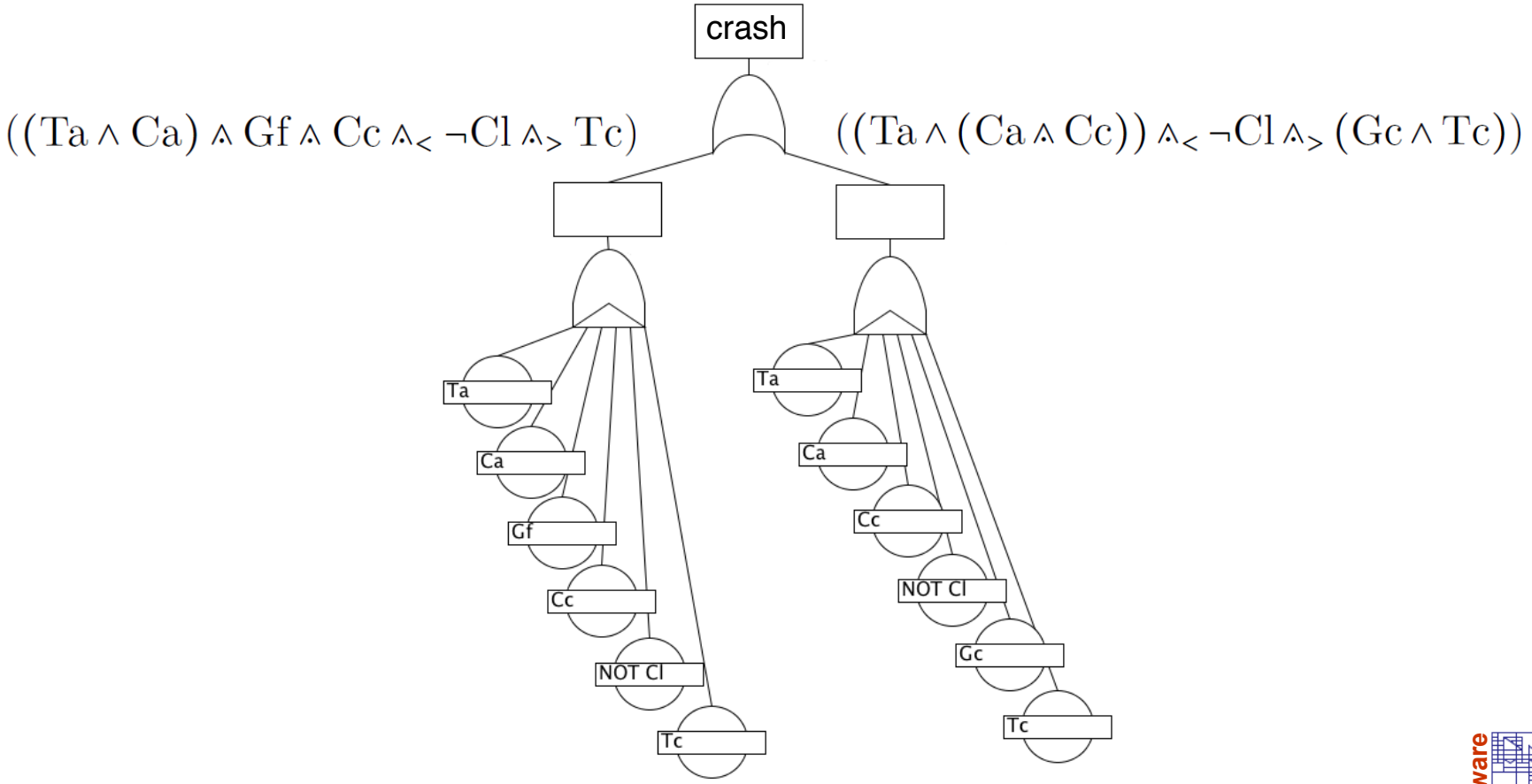
Tool Environment

- **QuantUM Tool Architecture**
  - prototypical implementation: SpinCause



Experiments

- **Railroad Crossing**
  - Promela model with 133 states and 137 transitions
  - represented as Dynamic Fault Tree



Experiments

- **Railway Crossing**

- **Airbag System**

- **Observations**
  - BFS outperforms DFS
    - rely on minimality of length of bad traces found
      - requires less good traces to be stored
  - on-the-fly outperforms off-line enumeration
    - on-line: only store red and black traces
    - off-line: store all traces

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Conclusion

- **Causality Checking**
  - technique complementing model checking
    - aim: algorithmic support for the debugging of models
  - defined / adopted causality model
  - proposed implementation
  - applicability to non-trivial case studies

- **Future Work**
  - causality checking at the limits of scalability
    - dealing with incomplete information
  - causality checking in a symbolic environment
  - on-line causality checking for probabilistic models
  - specific adaptions to functional safety analysis
    - minimal cut sets
    - root, common and cascading causes